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This research has been concerned with the development of a theory of control and observation for systems with time delays in the state, input variables. A theory, almost as complete for the non-delay situation has been constructed. The roles of stabilizability, reachability and observability concepts in this theory have been clarified. Numerical procedures for solving some of these problems have been developed.

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SYSTEMS WITH MEMORY AND SYSTEMS
DEFINED ON ALGEBRAIC STRUCTURES

FINAL REPORT

PROFESSOR SANJOY K. MITTER

JANUARY 27, 1978

Mr. Richard O. Ulsh, Chief
Information Processing Officer
Department of the Army
US Army Research Office
PO Box 12211
Research Triangle Park, NC 27709

DAAG29-76-C-0008

Massachusetts Institute of Technology
77 Massachusetts Avenue
Cambridge, Massachusetts 02139

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I. INTRODUCTION AND OBJECTIVES OF RESEARCH

The objective of this research is to develop a theory of control and observation for linear dynamical systems when there are time-delays present in the inputs (control) and observation. The basic model then is given by the stochastic differential equations

$$dx_t = Ax_t dt + \sum_{i=0}^{N} B_i u_{t+\theta_i} dt + d\xi_t \tag{1}$$

$$dz_t = Cx_t dt + \sum_{i=0}^{M} D_i x_{t+\alpha_i} dt + d\eta_t \tag{2}$$

where $\theta_N < \theta_{N-1} < \ldots < \theta_0 = 0$ and $\alpha_M < \alpha_{M-1} < \ldots < \alpha_0 = 0$, $x_t \in R^n$, $u_t \in R^m$, A, B_i, C, D_i are matrices of appropriate dimension and ξ_t and η_t are standard Wiener processes. (There are many variants of this basic model.)

The problems we wish to solve are:

(i) Let $\hat{x}_t = E(x_t | Z_t)$, where Z_t = σ-algebra generated by $\{z_\tau | 0 \leq \tau \leq t\}$. We wish to give a dynamical characterization of $\hat{x}_t$ in the form of a Kalman filter and study its asymptotic behavior.

(ii) Suppose we are required to choose the control (input) to minimize the performance function

$$J(u) = E\int_0^T \left[(x_t, Qx_t)_{R^n} + (u_t, Ru_t)\right] dt \tag{3}$$

where $Q \geq 0$ and $R > 0$. We would like to prove the so-called separation

theorem of stochastic control for this class of problems and study the asymptotic behavior of the resulting control system. Naturally, the last question requires introducing appropriate concepts of reachability and observability.

(iii) We wish to devise appropriate numerical schemes using finite element methods to solve such problems.

(iv) We wish to obtain a theory of realization for systems with time delay.

II. BACKGROUND OF THIS RESEARCH

In a series of papers (cf. [1], [2], [3], [4], [5]), we have developed a theory of functional differential equations, their control and state estimation for functional differential equations. A comprehensive account of this work will be available in the forthcoming monograph [6] (partially supported by the current grant). The major question that was not considered in this work was the presence of pure time delays in the input and output variables. The solution of problems described in Section I, when combined with that of my previous work will lead to a comprehensive theory of control and observation for functional differential equations in the presence of time-delays in the input and output variables.

III. DESCRIPTION OF CURRENT RESEARCH

(i) In the case where there are no delays present in the input variables, we can give a complete solution to the filtering and control problem. Results on this were previously obtained by Kleinman [7]. Our own work is an extension of these results. At the same time, our work is mathematically rigorous.

(ii) We have been able to solve the filtering problem for general linear hereditary systems with pure time delays in the observation process using probabilistic methods in a Hilbert space. It is conjectured that the control problem with pure time delays in the control variable can be solved using duality arguments. An account of these results will be available in [6] which has been submitted for publication to MIT Press.

(iii) In previous work, we have developed a duality theory (in the sense of mathematical programming) [8] for optimal control problems in the presence of control and state variable inequality constraints. We have extended this work to functional differential equations and have also extended our work on Ritz-Trefftz methods ([9],[10]) for the solution of such problems.

(iv) In cooperation with Prof. P. Fuhrmann of Israel, a theory of realization for delay systems has been initiated.

IV. POTENTIAL APPLICATIONS

There are several potential applications of this work. One area of application is as a model for human operators. Another possible application area is array-processing problems. Models in which the time-delay is a random variable may be of use in propagation problems through random media.

V. CONCLUSIONS

We now have a theory of linear time delay systems which is as complete as the theory of finite dimensional linear systems.

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VII. Personnel Supported by this Grant

1. Professor S.K. Mitter (Principal Investigator)

2. Professor P. Fuhrmann (Visiting Scientist, Summer 1977)

3. Mr. P. Toldalagi (Graduate Student)

VIII. Appendices

In Appendix 1, an abstract of a paper supported by the grant is described.

In Appendix 2, detailed contents of a forthcoming monograph is presented. The work leading to the monograph was partially supported by this grant.

APPENDIX I

Simulation of Linear Systems and Factorization of Matrix Polynomials*

by

PAUL A. FUHRMANN

Department of Mathematics
Ben Gurion University of the Negev
Beer Sheva, Israel

* This research was done while the author was visiting the Electronic Systems Laboratory at the Massachusetts Institute of Technology in Cambridge, Massachusetts 02139. It was supported by the Department of the Army under contract grant DAAG29-76-0008.

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1. Introduction.

This paper continues the investigations [1,2] into various aspects of linear system theory. As in the other papers the fundamental idea is the notion of a model of a linear transformation which is similar to the original but in some ways easier to handle.

It is quite well known that given a linear transformation acting as a linear space X over the field F then an $F[\lambda]$-module structure can be induced on X by letting $p \cdot x = p(A)x$ for all $p \in F(\lambda)$ and $x \in X$. Of course the action of A is identical to that of the polynomial $\chi(\lambda) = \lambda$ and the module is a finitely generated torsion module. The model approach reverses this approach. We start with an $F[\lambda]$-module X and define a linear transformation A in X by $Ax = \lambda \cdot x$. An interesting theory might arise if our choice of module X is well made. As our interest here is strictly in finite dimensional phenomena then we should restrict ourselves to finitely generated torsion modules over $F[\lambda]$.

From a system theoretic point of view there are two natural choices for our modules. It has been recognized by Kalman [5,6] that given a restricted input/output map f: $U[\lambda] \to \lambda^{-1}Y[[\lambda^{-1}]]$ then natural choices for a state space realization would be $U[\lambda]/\mathrm{Ker}\, f$ and Range f. The development in [1,2] used the first type of representation whereas here we investigate the dual representation using submodules of $\lambda^{-1}Y[[\lambda^{-1}]]$ as well as the relations between the two types of models. While $F[\lambda]$-module homomorphism in $U[\lambda]$ are easy to describe the module $\lambda^{-1}Y[[\lambda^{-1}]]$ is too big for a simple description of all $F[\lambda]$-homomorphism. However we obtain a certain lifting theorem, Theorem 2.4, which is sufficient for the application to system theory.

In section 3 we introduce a partial order into the sets of rational transfer functions and restricted input/output maps. This is the problem of when one canonical system can be simulated by another canonical system. For further back-

ground and results on this problem one should refer to [5]. Finally in the last section we apply the results on simulation to the problem of factoring a monic polynomial matrix into monic factors. This gives a system theoretic approach to some results of [3], which has the advantage that it holds over every field F. For further results and references on factorizations of matrix polynomials one should consult [7].

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APPENDIX II

Representation and Control of Linear Infinite Dimensional Systems

by

A. Bensoussan
M. C. Delfour
S. K. Mitter

(Forthcoming book which is now being considered for publication by MIT Press.)

Objective and Scope of the Book

The objective of this book is to give a coherent account of recent developments in system theory for linear infinite dimensional systems.

We feel that some of the principal contributions of system theory for finite-dimensional linear systems have been the following:

(1) Development of concepts of controllability and observability and its relation to stability and stabilizability. The main ideas here are due to R.E. Kalman. The importance of these ideas are that it leads to a qualitative theory of control and observation. In deterministic situations they allow us to conclude the existence of a stable controller and estimator which when combined gives rise to a stable compensator for the system.

(2) An investigation of the precise relationship between the input-output and state-space points of view for linear systems. The main topic here is realization theory and the main result is the construction of a canonical state space realization from a given input-output map. A canonical realization is one which is both controllable and observable. The main ideas here are again due to Kalman and the main tools are algebraic (module theoretic) in nature.

(3) Quadratic Cost Optimal Control Problems on a finite and infinite time interval and the development of a feedback solution to the control problem. One way of solving this problem is to use the ideas of Invariant Embedding as originally introduced by Bellman. The infinite-time problem requires the concepts of controllability and observability (or their weakening to stabilizability and detectability) to prove existence of a constant feed-

back solution. The method of proof is the use of Quasi-linearization. Here again the main result is a constructive way of obtaining a stable closed-loop system.

It is these developments which we propose to generalize to infinite-dimensional systems. We have in mind applications to the study of control of parabolic and hyperbolic partial differential equations and hereditary systems. This book might be considered to complement and add to the pioneering effort of J.L. Lions entitled "Optimal Control of Systems Governed by Partial Differential Equations", English Translation by S.K. Mitter, Springer-Verlag, Berlin, 1971.

In this book we study the issue raised in (1) and (3). Realization theory will be studied in a separate monograph by the authors.

This book consists of nine chapters. Chapters 0-3 lays the basic groundwork of the book. In Chapter 1 the mathematical preliminaries necessary to study the book are set out. Proof of results are generally omitted but precise references are provided. Chapters 2 and 3 describe the mathematical model of the systems whose control is studied in later chapters. Chapter 3 on Hereditary systems is mostly original work of the authors.

Chapters 4 and 5 discuss the main qualitative concepts of stability, stabilizability, detectability, reachability and observability in an abstract setting. They are then illustrated with reference to the concrete models described in Chapters 2 and 3. These results are needed to study the quadratic cost problem over an infinite-time horizon (cf. Chapter 7).

In Chapter 6 the feedback synthesis problem for linear systems with a quadratic cost function is studied. The problem is studied in an abstract setting which covers, for example, boundary control problem for distributed

systems as well as control problems involving delays in control and state variables.

In Chapter 7 the quadratic cost problem over an infinite time horizon is studied. A detailed study of the Operator Riccati equation is made and stability of the closed-loop regulator is proved. Much of this work represents original work of the authors.

It is well known that there is a duality between estimation and control. What does not seem to be well known is that these two problems lead to variational problems that are dual to each other. There is a long history of dual variational problems beginning with the Legendre Transform and the original work of Courant and Friedrichs. These issues are discussed in Chapter 8.

In Chapter 9 we apply the results obtained in previous chapters to study systems with time delay or more general hereditary systems. It is shown that by exploiting the structure of hereditary systems, detailed results for the quadratic cost problem can be obtained. Moreover, one can now give testable conditions for stabilizability and detectability. Systems with time delay are very important in practice. Their mathematical structure is also very interesting.

In summary, it is felt that the contributions of the proposed book are the following:

(1) It gives detailed results on representation theory and qualitative properties of systems which are important for the study of hereditary systems and distributed parameter systems.

(2) It gives a detailed theory of quadratic variational problems over a finite and infinite time interval.

(3) It shows how the theory can be applied to hereditary systems.

An introductory chapter will be written which summarizes the contents of the book, shows how the finite dimensional results have been generalized and also explains how the various parts of the book are related to each other.

This book should be of interest to system and control theorists and applied mathematicians. It is basically a research monograph but more or less self-contained and hence can be used as a text in a graduate course on Infinite Dimensional Systems.